TITLE: Uncertainty Quantification in Airframe Design

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Ramana V. Grandhi
Distinguished Professor
209 Russ Engineering Center
Department of Mechanical and Materials Engineering
Wright State University
Dayton, OH 45435, USA

rgrandhi@cs.wright.edu

Abstract

Multiple configurations in various stages of aircraft design have to be experimentally tested and validated to study the performance of various systems subjected to non-deterministic design parameters. These tests are expensive and time consuming, increasing the acquisition cost and time for military aircraft/equipment. Therefore, analytical certification aims at reducing/eliminating the expensive prototype testing during these intermediate design stages by propagating the input variance through the design. Analytical certification involves modeling the variance/uncertainties in the design parameters and estimating the variance in the component/system performance.

Based on the nature and extent of uncertainty existing in an engineering system, different approaches can be used for uncertainty propagation. If the uncertainty of the system is due to imprecise information and lack of statistical data, the Possibilistic theory can be used. During preliminary design, uncertainties need to be accounted for and due to lack of sufficient information assigning a probability distribution may not be possible. Moreover, the flight conditions (loads, control surface settings, etc.) during a mission could take values within certain bounds, which do not follow any particular pattern. The uncertain information in these cases is available as intervals with lower and upper limits. In this case, the fuzzy arithmetic based method is suitable to estimate the possibility of failure. The use of surrogate models to improve the efficiency of prediction is presented in this paper. Various numerical examples are presented to demonstrate the applicability of the method to practical problems.

Introduction

Aerospace structural design involves analyzing various disciplines like structures, controls, aerodynamics, aeroelasticity, electromagnetics, etc. The design process requires integration of multiple disciplines to model the actual behavior of the system. This complex design process usually becomes challenging with the presence of uncertainties in material properties, loads, boundary conditions, geometric properties, manufacturing processes, environment, and mathematical models. Incorporation of these uncertainties into the design enables the prediction of the aircraft performance variation in the presence of uncertainties and more importantly their sensitivity for targeted testing and quality control.

The traditional way to propagate the uncertainty is to use safety factors in the design, which essentially ignores the source of a given uncertainty. This safety factor approach produces designs that satisfy all the requirements but not optimum for the given conditions. Moreover, the safety factor based approach is suitable and applicable for situations where the new design is a derivative of an existing design. It becomes impractical and expensive to apply the safety factor based design to new systems because this procedure requires extensive testing to assign a safety factor that produces a conservative design.

As the complexity of the multifunctional structure increases, the cost of manufacturing the prototype to validate the designs and determine the level of safety would increase alarmingly. Thus, the need for analytical certification of components is compelling. Uncertainty quantification techniques are tools that aid in analytical certification. Uncertainty quantification and their effects are not unique. They depend on the amount of data engineers incorporate about a particular event or variable. Depending on the

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information available, the designer has to choose the quantification technique that would propagate the uncertainties through the system. Depending on the nature of information available, various techniques are appropriate for propagating the uncertainty through the system integration and design process.

In engineering design, the designers often have to deal with uncertainties in structural loads, material properties, initial conditions, control system settings, etc. The uncertainty can be classified as random and non-random. The random uncertainty can be dealt with by using the existing probabilistic methods and the safety of the design can be quantified [1-6]. In situations where the information required for assigning the probability distribution to a variable is not available, the non-randomness comes into picture. The impreciseness of the parameter is available as a bound. Various interval analysis techniques are available to deal with variables that have bounds. However, these techniques provide one bound for the response compared to the fuzzy approach that provides the bounds on the response at various confidence intervals. From the literature available on fuzzy arithmetic approach, fuzzy set models, which require little data, appear to be well suited to deal with design under uncertainty when little is known about the uncertainty.

Interval data might be available from the sparse output of instrument measurements; the mean of a normal distribution could be available from many experts; parameters of another distribution fit with high precision from a large collection of measured point data; and finally, the mean of another distribution may only be presumed to lie within an interval. Also, in real situations, the uncertainty in a given input parameter might be independently estimated from several completely different sources and thus have completely different mathematical representations.

Fuzzy theory provides a means by which incomplete or subjective information can be represented in an analytical form. This kind of uncertainty can arise during design and manufacture, where a geometric parameter, x can be subjected to tolerances as $x \pm \Delta x$. Moreover, no additional information is available to assign a probability distribution within the interval. Then the parameter can be treated as a fuzzy number. The main advantage of the fuzzy theory is that it can accommodate the confidence levels of variables and as the design progresses, the design need not be reevaluated to obtain the new bounds on the response due to change in confidence levels of the design parameters. These bounds will be available for any confidence level from 0 to 1, once the design is analyzed using the fuzzy theory. The technique used to construct the fuzzy number for the uncertain variable is discussed in the later section.

In 1965 [7] Zadeh provided the first tools, i.e. fuzzy sets, specially devised for dealing with vagueness. Since then various researchers have advanced the subject and its recent applications are in areas like artificial intelligence, image processing, speech recognition, biological and medical sciences, decision theory, economics, geography, sociology, psychology, linguistics and semiotics. Literature has shown that it is indeed a useful tool to quantify the impreciseness and vagueness present in real-life problems. Most of the engineering applications have been in controls, decision-making and optimization. Kaufmann and Gupta in Ref [8] reviewed the area of fuzzy arithmetic. Buckley and Qu [9], Sanchez [10], and Zhao and Govind [11] investigated the mathematics of fuzzy equations and their solutions. These solution methods are applicable to problems with explicit response functions and inapplicable when the response is implicit. Vallipapan and Pham [12] introduced the use of fuzzy information in the finite element analysis of geotechnical engineering application. In their work, the authors used fuzzy sets in the finite element formulation to model the elastic soil medium. They introduced the lower and upper fuzzy bounds values for the input parameters at a particular membership level, or α cut, and solved the resulting deterministic model.

Fuzzy finite element analysis can be broadly classified into two categories, namely, explicit and implicit techniques. In explicit techniques, the solution of the response is explicitly obtained by operations on an α level representation. The advantage of this method is that the interval equations as a function of α are solved only once and the bounds on the response at any α level are readily available. However, this method has its practical problems including the possibility of obtaining unbounded, unrealistic and non-unique solutions to the response bounds. Moreover, these methods are still being developed and they have not been well tested since they do not use the legacy finite element software.

The second category is the implicit formulation where all the binary combinations of the extreme values of the fuzzy variables at a particular α level are explored and the bounds of the response are obtained. At each combination of the variables, one finite element simulation is required and the

computational effort involved is exponentially related to the number of variables. The implicit methods result in exact bounds (provided there is no maxima or minima within the bound) at a considerable cost.

Braibant et al [13] presented non-deterministic possibilistic approaches for structural analysis and optimization. If the system is fuzzy, it is possible to establish a connection between the interval method and the fuzzy analysis by using the concept of membership level or α-cuts. α is actually the level of satisfaction, which changes from 0 to 1. At the level of satisfaction α, the variation domain of the variable x is given by the interval $[\, x_{\alpha}, x_{\alpha}\,]$. By representing the design parameters using α-cuts approach, all the interval expressions involved in the analysis can be evaluated at different fuzzy levels or α values. In practice, the fuzzy response of a structure is computed in three steps. First, the fuzzy members describing the parameter uncertainties are sampled for different degrees of membership in which each parameter is given an interval. This is what is called "fuzzification". Second, the finite element equilibrium equations are solved at each level, leading to the corresponding variation intervals of the structural responses. Finally, putting the intervals together for each structural response, the interval related to different degrees of membership allows the fuzzy response to be built. Difficulty arises when solving the discretized interval equilibrium equations.

In order to improve the efficiency of the above solution algorithms involving fuzzy data, the Hansen Algorithm [14], Neumann Approximate Vertex Solution and Vertex Solution [15] were introduced in Ref. [13]. The Vertex Solution is considered as the most robust but the computation cost could be prohibitive when the number of uncertain parameters is very large. The Hansen Algorithm is one of the most popular explicit direct algorithms and the iterative algorithms of the Gauss-Seidel or Jacobi family have to be used as a basis to solve the interval linear equation system. For the Hansen Algorithm, two modifications were introduced to limit the occurrence of unbounded solutions in the basic algorithm. These methods are applicable for implicit problems whose response behaves in a linear fashion. However, in most engineering problems, the response is highly nonlinear and a linear approximation at the central values could lead to erroneous results. For the Neumann Approximate Vertex Solution, the linear approximation is used for the stiffness matrix and the load vector with respect to uncertain parameters. The number of numerical simulations for Neumann is $2n$, where n is the number of uncertain variables. However, the "Vertex Solution" requires 2^n number of simulations. The cost involved in vertex method is exponential and it is not a viable solution to large-scale engineering problems.

Akpan [17] et.al presented response surface based fuzzy analysis. In their work, they have constructed a second-order response surface model and used this response surface to evaluate the function value in the vertex method. This method is applicable to problems with a limited number of uncertain variables since the cost of building the response surface increases with the number of uncertain variables. Moreover, the vertex method fails to capture the accurate bound when the function has maxima or minima within the range of the input parameters.

The above-mentioned difficulties in dealing with the fuzzy based analysis of structural systems are dealt with in the current study. In this current study, fuzzy set theory is applied to quantify the non-random uncertainties. The ability to efficiently handle large-scale implicit problems with high degree of accuracy is a salient feature of this method. The imprecision or fuzziness in the response of interest is calculated by using the Zadeh's extension principle. The computational implementation of the extension principle on implicit functions like a Finite Element Analysis (FEA) model or the Computational Fluid Dynamics (CFD) model is performed with the use of high quality function approximations. These function approximations reduce the cost involved in actual function value evaluations. Examples are provided to demonstrate the solution procedure and the applicability of the method.

Using the definition of the fuzzy number, the available uncertain information can be used to construct the fuzzy number. The present method is used to model the uncertainty when the available information about the uncertain variable is limited. The following section discusses the technique used to model the fuzzy number using the available information.

Triangular Membership Function

In structural engineering it is often possible to acquire knowledge about various parameters in the form of low, probable and high values. Based on this information, the membership functions can be constructed. Following the concepts of fuzzy set theory the parameters are modeled as fuzzy numbers,

where the information is imprecise due to vagueness. In this work we have adopted the suggestion found in Ref [12] to define a linear membership function.

The two extreme left and right fuzzy members L' and H' respectively, at $\mu(x)=0$ are defined as

$$L' = \begin{cases} P-2(P-L), & P \geq 2(P-L) \\ 0, & P \leq 2(P-L) \end{cases} \tag{5}$$

$$H' = P + 2(H-P) \tag{6}$$

where L, P, and H are the expert's estimates of low, probable and high values, respectively.

The following triangular membership function is obtained:

$$\mu(x) = \begin{cases} 0 & x \leq L' \\ \dfrac{x-L'}{P-L'} & L' \leq x \leq P \\ \dfrac{H'-x}{H'-P} & P \leq x \leq H' \\ 0 & x \geq H' \end{cases} \tag{7}$$

Several different shapes of membership can be used for different types of imprecision. This methodology permits the solution of problems involving imprecisely defined geometry, external loads, initial conditions, etc.

Multi-Point Approximation

The approximation technique used in this paper is the Multi-Point Approximation (MPA) technique. The multi-point approximation can be regarded as the connection of many local approximations. With function and sensitivity information of limit-state already available at a series of points, one local approximation is built at each point. All local approximations are then integrated into a multi-point approximation by the use of a weighting function. The MPA can be written using the following general formulation:

$$\tilde{F}(X) = \sum_{k=1}^{K} W_k(X)\tilde{F}_k(X) \tag{8}$$

where k is the number of local approximations, $\tilde{F}_k(X)$ is a two-point local approximation and W_k is a weighting function that adjusts the contribution of $\tilde{F}_k(X)$ to $\tilde{F}(X)$ in Eq. 8. The evaluation of this weighting function involves the selection of a blending function and a power index "m". The procedural details for evaluating the weighting function are discussed in Ref.18.

The weighting function is given by the equation

$$W_k(\mathbf{X}) = \frac{\phi_k(\mathbf{X})}{\sum_{j=1}^{K} \phi_j(\mathbf{X})} \tag{9}$$

where $\phi_k(X)$ is the blending function. The blending function used in this paper is given by:

$$\phi_k(\mathbf{X}) = \frac{1}{h_k} \tag{10}$$

This blending function combines the local approximations into one MPA and controls how fast the MPA adapts to the local approximation at a particular design point. The local approximations considered in this paper are TANA2 and they are of the type,

$$\tilde{F}(X) = F(X_2) + \sum_{i=1}^{n} \frac{\partial F(X_2)}{\partial x_i} \frac{x_{i,2}^{1-p_i}}{p_i} (x_i^{p_i} - x_{i,2}^{p_i}) + \frac{1}{2}\varepsilon \sum_{i=1}^{n} (x_i^{p_i} - x_{i,2}^{p_i})^2 \tag{11}$$

where X_2 is the expansion point for the approximation. X_1 and X_2 are two design vectors at which the function and gradient information of F(X) are used to build TANA2 model. This equation is a second-order Taylor series expansion in terms of the intervening variables $y_i (y_i = x_i^{p_i})$, in which the Hessian matrix has only diagonal elements of the same value ε. Therefore, this approximation does not need the calculation of the second-order derivatives.

The construction of MPA requires sampling (D-Optimal design) in the entire domain represented by the uncertain variables. Design of experiments technique is used to efficiently select the data points required for the construction of MPA.

These types of designs are always an option regardless of the type of model the experimenter wishes to fit (for example, first order, first order plus some interactions, full quadratic, cubic, etc.) or the objective specified for the experiment (for example, screening, response surface, etc.). Given the total number of treatment runs for an experiment and a specified model, the computer algorithm chooses the optimal set of design runs from a *candidate set* of possible design treatment runs. This candidate set of treatment runs usually consists of all possible combinations of various factor levels that one wishes to use in the experiment.

Uncertainty Quantification Method

The problem dealt with here is the estimation of the membership function of the response subject to uncertain input parameters. The uncertainty is non-random and it is defined using the fuzzy set theory. The estimation of the fuzzy membership function for the implicit response requires the use of interval analysis at each α level. The interval analysis techniques available in the literature as discussed earlier use linear approximation at the central values or evaluate the function at all the vertices formed by the lower and upper limits of the uncertain variables. These methods require a significant number of function evaluations and sometimes fail to capture the bounds of response for non-monotonic response functions. Therefore, a method using the nonlinear function approximations to reduce the computational effort involved in the analysis is presented. The UQ method has two main tasks: (1) Fuzzification; and (2) Computation of fuzzy response based on extension principle. These are discussed below:

Fuzzification

A fuzzy set is an imprecisely defined set without a crisp boundary and it provides a gradual transition from 'belonging' to 'not belonging' to the set. The process of quantifying a fuzzy variable is known as fuzzification. When the input parameters are uncertain then they have to be fuzzified. This is done by constructing a membership function (possibility distribution) for the variable. In the present work, convex normal triangular membership functions are used to characterize the fuzzy input variables.

Fuzzy Membership Function Estimation

In order to employ the extension principle, the membership function of the response is obtained from the surrogate model of the response, using multi-point approximations. Then this approximation is used along with the numerical estimation method for estimating the membership function of the response.

The following are the main steps involved in the approximation of the response

1. Use design of experiments to obtain the location of design points that are used to approximate the response

2. Perform actual analysis and obtain the function value and gradients of the response at the above selected design points
3. Construct TANA2 at the design points and blend them into a single approximation using the Multi-Point Approximation (MPA) techniques

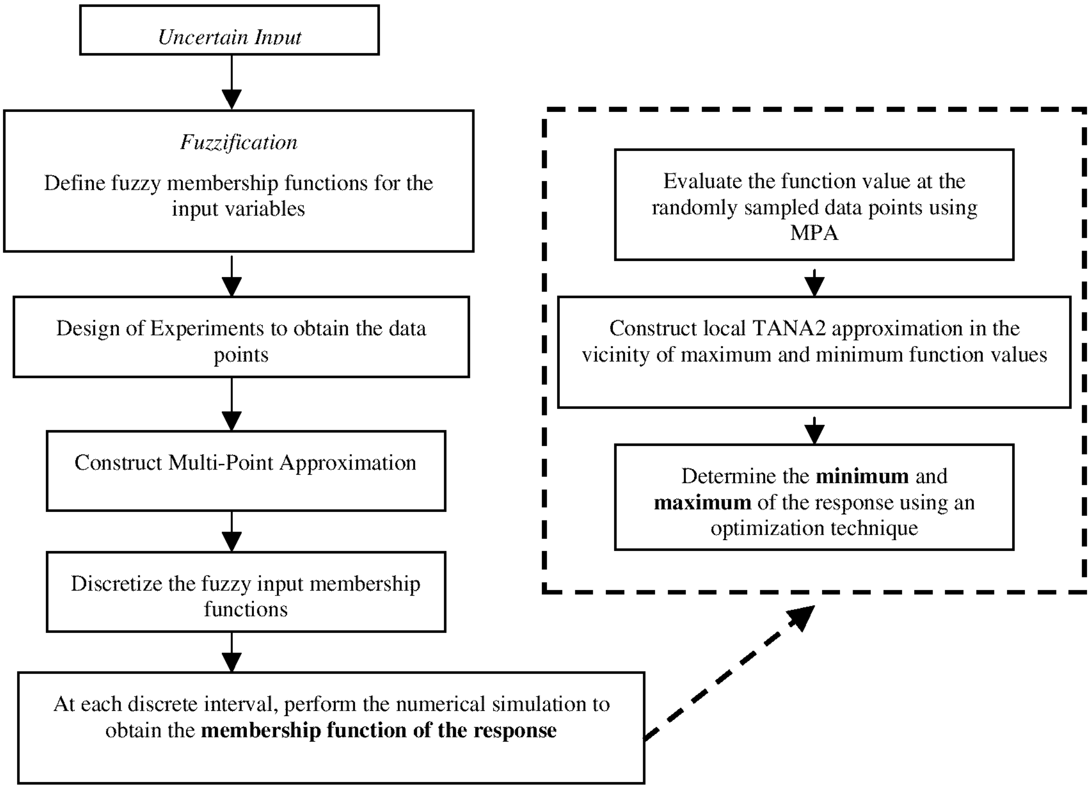

Figure 1 Possibilistic Structural Analysis Algorithm

Once the fuzzy input variables and the approximate response function are available, the vertex method can be used on the approximate response function to obtain the bounds on the response. However, the vertex method is computationally expensive and does not produce accurate results when the response is non-monotonic. Therefore, a more robust method to accurately estimate the bounds of the response at different possibility levels is presented in this work.

The following are the steps involved in estimation of the bounds on the response:

1. Discretize the membership function of the input parameters at various α cuts (possibility levels). At each of these discretized level the maximum and minimum of the response are calculated to construct the membership function of the response.
2. Evaluate the approximate function value at randomly sampled data points for each discretized α level. The data point that results in a minimum function value is used as a starting point of the optimization process. The data points are selected by using uniform distribution for sampling. This assumption of uniform distribution does not affect the final result because it is used only to sample the data. Use of a different distribution would assume bias towards a certain region in the entire design space and would result in erroneous results.
3. Use TANA2 approximation during the search process to reduce the computational cost. The TANA2 approximation is constructed using new data points in the vicinity of the data point determined in step 2.
4. Repeat step 2 to obtain a maximum value of the response by constructing new TANA2 approximation.

Once the minimum and maximum values of the function are obtained, the membership function of the response can be determined. Using this membership function the possibility of failure for the structure can be determined using the possibility theory. Figure 1is a pictorial representation of the UQ method. The

process in the dotted lines, in figure 1 is the numerical technique used to determine the membership function of the response.

Flexible Wing Example

Figure 2 shows the structural model of the flexible wing whose membership function for the frequency response is considered. The structure represents a fighter wing model with all the dynamic characteristics. There are 118 nodes in the wing section, 12 nodes in fuselage part and one reference node. The connection of the wing to the fuselage is modeled using simple beam elements. The upper and lower skins are modeled using quadrilateral and triangle membrane elements. The rib and spar webs are represented by shear panel elements, while the rib and spar caps are represented by rod elements. The vertical stiffness is supported by rod elements.

The wing structure is composed of 562 elements, which are classified as skins, ribs, spars, rib caps, spar caps, and posts. The weight of the whole fuselage is 16,000 lb and due to symmetry, each wing carries 8,000 lb. In addition to the load, 1,600 lb of nonstructural mass is distributed among the free nodes. First natural frequency of the structure is considered as the response in this example. The structure is analyzed using the finite element software ASTROS [20].

The response function considered in this example is as shown below:

$$G(X) = \frac{(2\,\pi\, f)^2}{\lambda_1} - 1$$

where f is the lower limit on the fundamental frequency that is 3.0 Hz. and λ_1 is the fundamental eigenvalue. The vector X represents all the uncertain variables defined as triangular membership functions. Table 1 shows the lower and upper limits of the uncertain variables along with the central value used to construct the triangular membership functions.

The uncertain variables are obtained by using physical linking of the design parameters in order to reduce the size of the 562 element problem. Physical linking facilitates the reduction of number of uncertain variables, so that manufacturing and assembly issues are represented in modeling. At the same time, the computational schemes can be applied to practical large-scale problems. Upper wing skin is modeled to have the chord wise element thickness as uncertain variable. This linking results in seven uncertain variables for the upper skin since there are seven rib sections excluding the root. The lower skin properties are matched to the properties of the upper skin. The upper and lower spar caps are assigned one uncertain variable. The leading edge spar web is assigned one uncertain variable and the upper and lower spar caps are assigned one variable. Seven variables are used to model the spars. There are seven rib sections that are assigned one uncertain variable each. The upper and lower rib caps are modeled as one variable. The smart actuators are modeled as one variable and the vertical stiffeners are modeled as another variable.

The values of the uncertain variables are taken from an optimization study. The optimization study was performed using the weight of the structure as the cost function and this is minimized subject to a frequency constraint. A lower limit of 3 Hz was used as a constraint on fundamental frequency. The structure was optimized with this frequency constraint and the final design is selected as the central design. The initial weight of the structure was 13,000 lb. and the final design had a weight of 9833 lb. Figure 3 shows the iteration history for the above optimization problem. Once the central design is selected, uncertainty is modeled as deviation from the central design. This type of modeling would be suitable when dealing with designs that have tolerance information.

D-optimal design technique was used to select 512 data points to construct MPA for the response function. Local TANA2 approximations were constructed using the data points and these were blended using a blending function to obtain one MPA. Once the approximation is constructed random sampling is performed in the uncertain space and the maximum and minimum for the function are determined. These maximum and minimum points are perturbed and new simulation is performed at the perturbed data points. This additional information is used in constructing a new TANA2 approximation that is used to determine the lower and upper limit of the response.

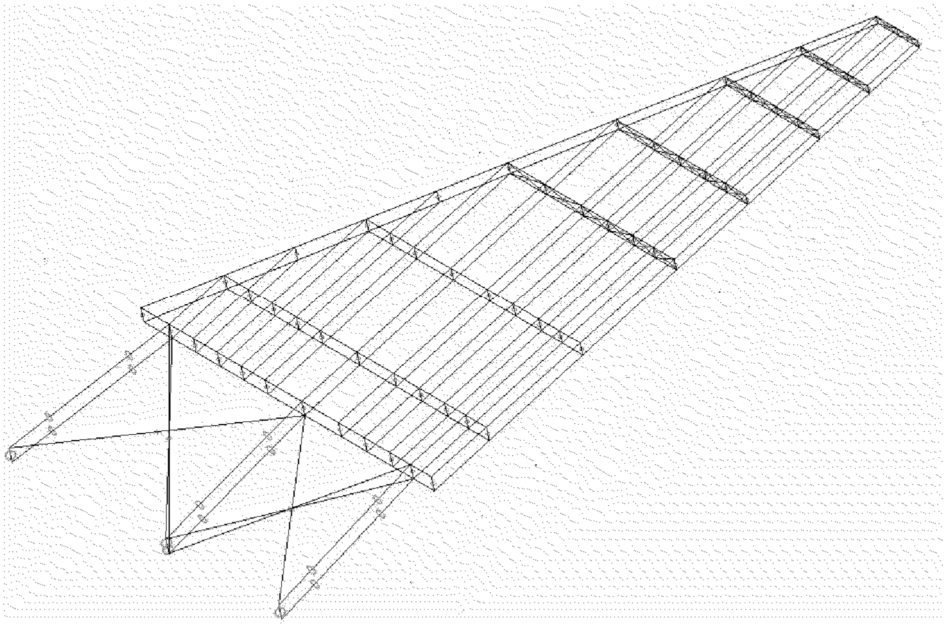

Figure 2. Flexible wing structural model

At every data point, the function value and gradients are estimated using ASTROS. The gradients are available analytical at every data point. A total of 572 simulations were performed to obtain the membership function of the response. Each of these simulations is a computationally expensive finite element analysis. Among these, 512 simulations are performed at each of the data points from the D-optimal design, 40 simulations were required to construct the TANA2 approximation, used in optimization to find the maximum and minimum. Finally 20 simulations were performed to obtain the actual function value at the optimum points for minimum and maximum values.

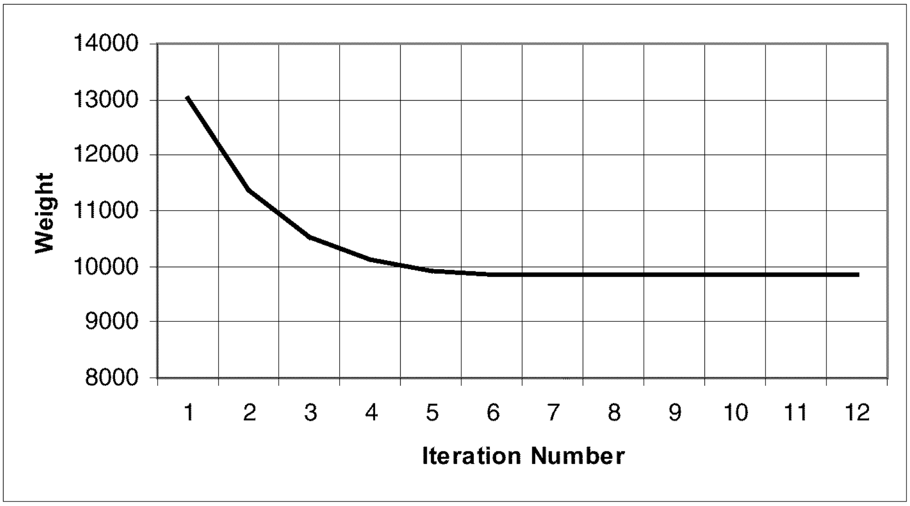

Figure 3: Iteration History

These lower and upper limits are obtained at each of the possibility levels to produce the membership function of the response as shown in Figure 4. The membership function determines the bounds on the response at various confidence levels. For example, the response value would be equal to –

0.00153 if the structural parameters were deterministic and assumed the central value. However, the uncertainty in the input data would result in the bounded response depending on the bounds of input parameters.

The possibility of failure is equal to the maximum value of the confidence level in the range of response values considered. For example, from the plot of the membership function of frequency, the possibility of failure that the frequency would be less than -0.1 is 47%. This is obtained by considering the interval from –0.1 to the lower limit when the confidence level is 0 and taking the value of the maximum confidence level for all those values. The possibilistic analysis method is an efficient technique to estimate the possibility of failure of the response. The efficiency of the method is achieved by using the high fidelity surrogate models for the response.

Uncertain Variable	**Lower Limit (in.)**	**Central Value (in.)**	**Upper Limit (in.)**
1	0.10	0.130	0.15
2	0.06	0.080	0.10
3	0.05	0.070	0.09
4	0.04	0.056	0.08
5	0.02	0.036	0.05
6	0.01	0.020	0.03
7	0.01	0.020	0.03
8	0.01	0.020	0.03
9	0.01	0.020	0.03
10	0.01	0.020	0.03
11	0.01	0.020	0.03
12	0.01	0.020	0.03
13	0.01	0.020	0.03
14	0.01	0.020	0.03
15	0.02	0.026	0.031
16	0.01	0.015	0.02
17	0.03	0.040	0.05
18	0.01	0.020	0.03
19	0.01	0.020	0.03
20	0.01	0.020	0.03
21	0.01	0.020	0.03
22	0.01	0.020	0.03
23	0.02	0.030	0.04
24	0.02	0.030	0.04
25	0.035	0.045	0.055
26	0.01	0.020	0.03
27	0.01	0.020	0.03

Table 1: Bounds on the Uncertain Structural Parameters

A value of possibility equal to zero means that there is no possibility and a value of one indicates maximum possibility. The above membership function describes the relationship between the possibility level and frequency. The membership function can give information about the possibility for a range of frequencies. In this technique, the structure can be designed to operate in the range of response values that satisfy certain confidence level requirements.

Therefore, when designing a structure such as this fighter wing, the designer must decide what level of confidence has to be achieved for the design. Once that confidence level is decided, the problem boils down to controlling the various uncertain parameters in order to obtain a required membership function. These uncertain parameters can be controlled by posing this as an optimization problem where the objective could be minimization of weight of the structure and the constraint is that the possibility of a particular value (say frequency less than a certain limiting value) is less than a predetermined level of confidence. This optimization task would produce a design whose frequency is within the acceptable range at a particular level of confidence and also the structure would have least weight.

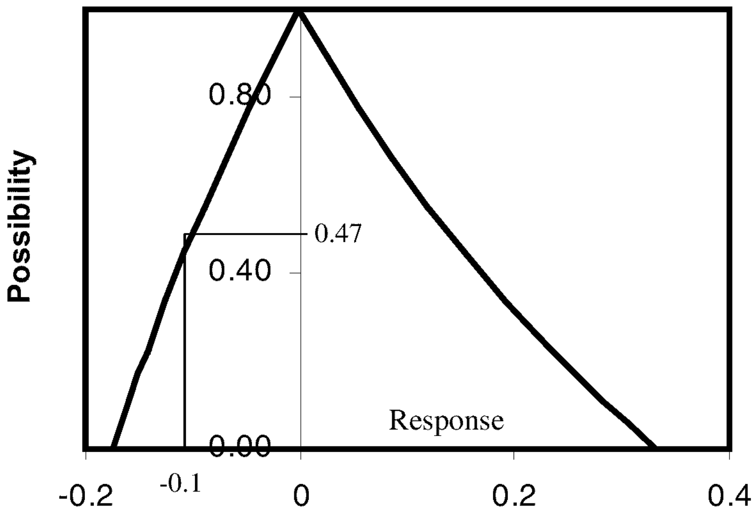

Figure 4: Bounds on Frequency Response

Summary

The uncertainty quantification method presented in this paper aims at reducing or even eliminating the testing on prototypes during the intermediate stages. These methods can be used to determine the variance in the response analytically and the design can be updated according to this information. This reduces the cost and time for military vehicle acquisition considerably. Moreover, these methods can also be used to develop a targeted testing procedure for the final design prototypes thereby, reducing the number of actual tests.

In this paper a methodology for dealing with vague information is presented. In the presence of vague information, usual practice is to assign some distribution information that closely matches the available information. However, this practice would introduce more uncertainty into the system and the results obtained can be orders of magnitude away from the actual failure probability.

Therefore, the presented method uses the available information without assuming any additional information and predicts the possibility distribution for the response. This possibilistic analysis is useful in the preliminary design stage where very less data is available for the design parameters. The efficiency of the UQ method is evident when dealing with the implicit response functions that require expensive FEA

simulations to obtain the function values. Therefore, the use of function approximations is emphasized to reduce the computational cost involved in the analysis procedure.

In possibility analysis a membership function has to be assigned for the uncertain variable. This membership function can be constructed when the intervals information at various confidence levels is known. However, this information is usually unavailable, therefore, a distribution is assumed that best represents the available data. In this paper, a triangular membership function is assigned to the uncertain variables because it is assumed that the data is dispersed around a central design. The membership function of the response depends on the assumption of a distribution for the individual uncertain variables. Therefore, assumption of an invalid distribution would produce results that do not represent the actual problem. However, the values of the response at zero percent confidence are not dependent on any kind of distribution information. Therefore, when there is absolutely no information about the uncertain variables the analysis is performed at zero percent confidence.

The possibilistic analysis method discussed here would estimate the bounds on the response subject to uncertain intervals. These bounds give an insight into the problem and they can be used to design structures that are less sensitive to the uncertainties in the input parameters. These bounds can be used in an analysis procedure that adjusts the bounds of input parameters to determine a configuration of these parameters that would result in the narrowest bound on the response. Moreover, the worst-case bounds on the response can be forced to fall in the safe operating zone to obtain a highly reliable component or system. These design configurations produce structures that are robust and perform safely in an uncertain operating environment.

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Paper #26

Discussor's Name: Steve Whittle
Author's Name: Ramana Grandhi

Q: Your presentation concentrated on how input uncertainty propagates through models to produce levels of uncertainty in the output properties. Equally important are levels of uncertainty in the model itself. Have you considered/completed model dependability analysis in you research?

A: Uncertainties in simulations come from many sources such as input parameters, finite element models, type of elements, numerical analyses selected, nonlinear solution techniques chosen, initial conditions, assumptions in solvers, and soon. This presentation concentrated on input parameters and we are in preliminary stages of research in model uncertainty and including them. The key factor is how to characterize model uncertainties and then choosing a proper method for propagation in simulation.